USMANU DANFODIYO UNIVERSITY, SOKOTO

(POSTGRADUATE SCHOOL)

EVALUATION OF RELIABILITY AND AVAILABILITY CHARACTERISTICS OF TWO DIFFERENT SYSTEMS USING LINEAR FIRST ORDER DIFFERENTIAL EQUATION

A Dissertation

Submitted to the

Postgraduate School

USMANU DANFODIYO UNIVERSITY, SOKOTO, NIGERIA

In Partial Fulfillment of the Requirements
For the Award of the Degree of
MASTER OF SCIENCE (MATHEMATICS)

By

AMINU, HARUNA
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DEPARTMENT OF MATHEMATICS

May, 2014

DEDICATION

This research work is dedicated to my father Alhaji Haruna Mohammed B/Yauri and my mother Hajiya Amina Haruna .

ACKNOWLEDGEMENTS

All praises are due to Allah, Most Gracious, Most Merciful for making this research work a reality. My sincere thanks go to my Supervisors Dr. U.A. Ali, Dr. A. Danbaba, and Dr. H.N. Yahya for their tremendous contribution and effort toward completion of this work. My profound gratitude goes to my father and my mother Alhaji Haruna Mohammed B/Y

Director Health and Hajiya Amina Haruna for their tremendous support financially may Allah reward them with jannatul Firdausi, amen. My special thanks go to the Head of Department of Mathematics, Professor, I.J. Uwanta.

My special thanks also go to Professor M. O. Ibrahim, Professor A. Bayoumi, Professor A. Ibrahim, Dr. B.A.Madu, Dr. A.I Garba, Dr. Sama'ila Kamba and all the staff of Mathematics Department. I will also like to extend my gratitude to my colleagues and course mate for their help and encouragement. Finally, I would like to thank my wife Jamila Idris Makawa for her encouragement, perseverance and continuous prayers throughout my programme.

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NOTATIONS/ ABBREVIATIONS

N : Normal State of unit.

Np : Normal unit under Preventive Maintenance.

$F_{1:}$ One kind of Failure State which causes self- reset of the failed unit.

F_2 ; The other kind of Ffailure State which leads to Maintenance.

E_j : Expected time to reach an absorbing State, j = 1,2.

$\lambda_{1\,;}$ Failure rate model $F_{1.}$

$\lambda_{2:}$ Failure rate model F_2 .

$\mu_{1;}$ Repair rate of each unit by self reset.

$\mu_{2:}$ Repair rate of each unit by Maintenance Facility.

$\mu_{3:}$ Repair rate when the System is in failed State.

u(t) : Pdf of time for taking a unit in to Preventive Maintenance

i.e, $u(t) = \theta_1(t) = \exp(-\theta_1 t), \theta_1, t > 0$

v(t) : Pdf and Preventive Maintenance time, $v(t) = \theta_2(t) = \exp(-\theta_2 t), \theta_2, t > 0$

$P_i(t)$: Probability that the System is in State S_i, i = 0,1,2,3,4,5,6,7.

P(t) : Probability Vector consisting of $p_i(t)$.

$S_{i:}$ Transition States, i = 0,1,2,3,4,5,6,7.

$A_{Tj}(\infty)$: System Steady-State Availability, j = 1,2

IRE: Institute of Radio Engineers

IEEE: Institute of Electrical and Electronic Engineers

SC: Station Computer

MMI: Man Machine Interface

SCADA: Supervisory Control and Data Acquisition System

MTSF: Mean Time to System Failure

MTBF: Mean Time between Failure

SSA: Steady- State Availability

BP: Busy Period Analysis

PF: Profit Function

LIST OF FIGURES

ABSTRACT

This study deals with the reliability and availability characteristics of two different systems, the second system differs from the first system due to the additional feature of preventive maintenance. Reliability and Availability analysis of system having one active unit and one warm stand-by unit with self-reset function and one maintenance facility. The failure unit is repaired through self-reset or maintenance according to different failure model.(Mean Time to System Failure), Steady- State Availability, Busy Period Analysis and Profit Function are derived for the two systems using linear first order differential equations. Two systems were evaluated theoretically and graphically to observe the effect of preventive maintenance on systems performance. The result finally shows that increase in failure rate leads to decrease in MTSF, Steady-State Availability and Profit Function of figure 4.1, 4.2 and 4.3. It was also found that increase in repair rate leads to increase in MTSF, Steady-State Availability and Profit Function of figure 4.4, 4.5, and 4.6. Therefore, the result indicated that second system originate better reliability due to the additional feature of preventive maintenance.

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CHAPTER ONE

1.0 GENERAL INTRODUCTION

1.1 Introduction:

The role and importance of reliability have been a core of any engineering industry for the last three decades. Reliability is of importance to both manufacturers and consumers. So, the reliability measure is very important, as the improvement of reliability is achieved through quality. While this measure of reliability assumes great importance in industry, there are many situations where continuous failure free performance of the system, though desirable may not be absolutely necessary, Yadavalli and Vanwyk (2012).

Several authors have studied a two (or more) similar and dissimilar unit standby redundant system. Haggag (2009a), studied the cost analysis of dissimilar-unit cold-standby system with three state and preventive maintenance using linear first order differential equations.

El-sherbeny et al (2009), studied the optimal system for series systems with warm standby components and a repairable service station. Researchers in reliability have shown a keen interest in the analysis of two (or more) component parallel system owing to their practical utility in modern industrial and technological set ups.

Two unit warm standby redundant systems have been investigated extensively in the past. The most general model is the one in which both the life time and repair time distributions of the units are arbitrary. However the study of standby system with more than two units, though very important, has received much less attention, possibly because of the built in difficulties in analyzing them. Such systems have been studied only when either the life time or the repair time is exponentially distributed. When both these are general, the problem appears to be intractable even in the case of cold standby systems. The present contribution is an improvement in the state of art in the sense that a three unit warm standby system is shown to be capable of comprehensive analysis. In particular we show that there are imbedded renewal points that render the analysis possible. Using these imbedded renewal points they obtained the reliability and availability functions, Srinivasan and Subramaniam (2006).

But In this research, the reliability and availability characteristics of two different systems are study, where the second system differs from the first system due to the additional feature of preventive maintenance. Each system consisting of one active unit and one warm standby unit with self-reset function and maintenance facility. The failure unit is repaired through self reset or maintenance according to different failure models.

1.2: Background to the Study

Reliability and availability are very important indices in a substation control protection system. The Station Computer (SC) has important role in the system. Its function is to maintain the central system data base and provide interfaces to the outsider world-locally to station operators through the local Man Machine Interfaces (MMI) subsystem and remotely to system operators and protection engineers through Supervisory Control and Data Acquisition system (SCADA) communication interfaces. So, its reliability directly influences the reliability of Station Computer (SC). Two units warm standby redundancy is taken. Redundancy is one of the ways of improving the reliability of system when the individual unit of the system remains unchanged. Warm standby is essential for two units to switch within the shortest time. So, the active unit and warm standby unit run in different states, which makes their failure rate different. Commonly, the failure rate of the warm standby unit is smaller than that of the active one. So, compared with a hot standby system, the reliability of the warm standby system is increased. Second, each unit has a self reset function. Each unit performs automatic error detection through self – checking and recovers from some failures, El-Said and El-Hamid (2006).

1.3: Reliability Measures

Reliability is the analysis of failures, their causes and consequences. It is the most important characteristics of product quality as things have to be working satisfactorily before considering other quality attributes. Usually, specific performance measures can be embedded in to reliability analysis by the fact that if the performance is bellow a certain level, a failure can be said to have occurred.

1.3.1: Reliability is the probability that the system will perform its intended function under specified working condition for a specified period of time. Mathematically, the reliability function R(t) is the probability that a system will be successfully operating without failure in the interval from time zero to time t.

$R(t) = P(T > t), t \geq o$

where T is a random variable representing the failure time or time – to failure. The failure probability, or unreliability is then $F(t) = 1- R(t), = P(T\leq t)$ which is known as the distribution function of T.

1.3.2: Mean Time to Failure the mean time to failure (MTTF) is defined as the expected value of the lifetime before a failure occurs.

1.3.3: Failure Rate Function and Repair Rate Function

The failure rate function, or hazard function, is very important in reliability analysis because it specifies the rate of the system aging.

The Failure Rate Function: Is defined as the quantity representing the probability that a device of age t will fail in the small interval from time t, to t + dt. The importance of failure rate function is that it indicates the changing rate in the aging behavior over the life of a population component.

Repair Rate Function: Is the expected time to repair the system from failure. This include the time it takes to diagnose the problem, the time it takes to get a repair technician on site, and the time it takes to physically repair the system, Pham (2003).

1.3.4: Maintainability and Availability

When a system fails to perform satisfactorily, repair is normally carried out to locate and correct the fault. The system is restored to operational effectiveness by making an adjustment or by replacing a component.

- Maintainability: Is defined as the probability that a failed system will be restored to a functioning state within a given period of time when maintenance is performed according to prescribed procedures and resources. Generally, maintainability is the probability of isolating and repairing a fault in a system within a given time. Maintenance personnel have to work with system designers to ensure that the system product can be maintained cost effectively.

- The Availability Function of a system, denoted by A(t) is defined as the probability that the system is available at time t. Different from the reliability that focuses on a period of time when the system is free of failures, availability concerns at a time point at which the system does not stay at the failed state.

Mathematically, A(t) = Pr(system is up or available at time instant t).

1.3.5: Mean Time to Failure (MTTF), and Mean Time Between Failure (MTBF) it is important to distinguish between the concepts mean time to failure and mean time between failures (MTBF). The MTTF is the expected time to failure of a component or system. That is, the mean of the time to failure (TTF) for that component or system. The MTBF is the expected time to failure after a failure and repair of the component or system.

1.3.6: Preventive Maintenance the maintenance carried out at predetermined intervals or corresponding to prescribed criteria and intended to reduce the probability of failure or the performance degradation of an item. Hoyland and Naws (1994).

1.4: Aim and Objectives

The main aim of this research work is to investigate and improve upon the existing methodologies for the reliability and availability characteristics of two different systems, where the second system differs from the first system due to the additional feature of preventive maintenance. To achieve the above aim the following objectives are derived.

- To observe the effect of failure rate, repair rate and preventive maintenance on both system, in terms of their MTSF, Steady-State Availability and Profit Function.

- To evaluate the MTSF, Steady-State Availability and Profit Function of the two systems theoretically and graphically and to also identify which originate better reliability and availability, due to the effect of preventive maintenance.
- To determine the busy period of the two systems.

1.5: Scope and Limitation the scope of this research is an investigation in to the Mean Time to System Failure, Steady State Availability, Profit function as well as Busy Period of two systems using linear first order differential equation. In which the result are finally evaluated, theoretically and graphically to observe the effect of preventive maintenance on systems performance.

Limitation: This research work is limited to Mean Time to System Failure, Steady State Availability, Profit Function and Busy Period with respect to Failure Rate and Repair Rate of two systems.

1.6 :Suggestion for Further Studies the work of this research shall be an extension of the systems. Since the research work is limited to only two systems, and then it can be extended to three systems with three states, Normal Unit, Partial Failure Unit and Total Failure Unit of Warm Standby. Supporting Unit and Preventive Maintenance.

CHAPTER TWO

2.0: LITERATURE REVIEW

Redundancy plays an important role in enhancing system reliability. Which redundancy has been analyzed for many difference system structures. One of the commonly used forms of the redundancy is the standby redundancy. Standby systems often find applications in various industrial and other set ups. In standby redundant system, some additional paths are created for proper functioning of the system. Standby unit is a support to increase the reliability of the system.

In practice, system do not always fail with major breakdown, it is developed that various mathematical models consisting of two types of failure: major and minor. A maintenance policy that suits a system presenting two types of failure represented by many earlier researchers revealed that minor failures are removed by a minor repair that brings the system back to the operating condition. A major repair is restored the system as good as new,Agarwal *et al* (2010).

2.1: Relationship between Availability, Reliability and Maintainability

Availability: Is defined as the probability that the system is operating properly when it is requested for use. In other words, availability is the probability that a system is not failed or undergoing a repair action when it needs to be used. At first glance, it might seem that if a system has a high availability then it should also have a high reliability. Reliability on the other hand represents the probability of components, parts and system to perform their required functions for a desired period of time without failure in specified environments with a desired confidence. Reliability, in it self does not account for any repair action that may take place. Reliability account for the time that it will take component, part or system to fail while it is operating. It does not reflect how long it will take to get the unit under repair back in to working condition. As stated, availability represents the probability that the system is capable of conducting its required function when it is called upon given that it is not failed or undergoing a repair action. Therefore not only availability a function of reliability, but it is also a function of maintainability, Liao *et al* (2006).

2.2: Availability Classification

Availability is classified by Nelson (1982) as follows:

- Point Availability: Point or instantaneous availability is the probability that a system (component) will be operational at any random time t.
- Mean Availability: The mean availability is the proportion of time during a mission or time-period that the system is available for use. It represents the mean value of the instantaneous availability function over the period (0, T).
- Steady State Availability: The steady state availability of the system is the limit of the instantaneous availability function as time approaches infinity. The instantaneous availability function approaches the steady value very closely at time approximate to four times the MTBF.
- Operational Availability: Is a measure of availability that indicates all the experienced sources of downtime, such as administrative downtime, logistic downtime etc, Nelson,(1982).

2.3: Standby Classification

Nelson (1972) classified the System Standby as follows

- A **Cold Standby System:** Is a redundancy method that involves having one systems as back up for another identical primary system . The cold standby system is called upon only a failure of the primary system.
- **A Warm Standby System:** Is a redundancy method that involves having one system running in the background of the identical primary system. The data is regularly mirrored to the secondary server. Therefore at times, the primary and secondary systems do contain different data or different versions. A warm server is turned on periodically to receive update from the warm standby machine.
- **A Hot Standby System:** Is running simultaneously with another identical primary system. On failure of the primary, the hot standby system immediately takes over to replace the primary, Nelson,(1972)

Agarwal et al (2010), studied the reliability characteristics of cold-standby redundant system. The objective is to improve the reliability of the system through append the redundant component. Two unit in cold-standby is considered. Each unit of the system has two modes Via; operable and failed. The failure of units are of two types: minor and major. After major failure, cold standby unit replaces the failed unit after a random amount of time. Where the failure and repair times follow exponential and general time distribution respectively. The basic equation has been transformed in to an integro-differential equation and solve it using supplementary variable techniques, various reliability parameters have been computed and analyzed by tabular and graphical illustrations

Qingtai and Shaomin (2011), studied the reliability analysis of two-unit cold standby repairable systems under poison shock. He analyzed the reliability of cold standby system consisting of two repairman. At any time, one of the two units is operating while the other is on cold standby. The repair man may not always at the job site, or take a vacation. They assume that shocks can attack the operating unit. The arrival times of the shocks follow homogeneous poison process and their magnitude is random variable following a known distribution. Where time on repairing a failed unit and the length of repair men's vacation follow general continuous probability distributions, respectively. They drive a number of reliability indices: System reliability, Mean time to First Failure, Steady-State Availability, and Steady-State Failure Frequency.

Two unit warm standby systems with preparation time for the repair facility. Anon – markovian model of two unit warm standby system with random preparation time for the repair facility after completion of each repair is considered. A unit while online has an arbitrary life time distribution and while in standby has a constant failure rate. The distribution of the preparation time for the repair facility is enlarging of order n. equation satisfied by the reliability and availability functions of the system are obtained. Meantime to system failure is derived, Yadavalli and Vanwyk (2012).

Mohammed (2012), studied the Cost benefit analysis of series systems with mixed standby components and K-Stage Erlang Repair time. They compare the availability characteristic between three different series system configurations with mixed (cold and warm), on the assumption that the time to failure for each of the operative and warm standby components follow exponentially distributed with parameters respectively. They present a recursive method, using the supplementary technique; developed the explicit expressions for the steady – state availability. Under the cost benefit criterion, comparisons are made based on assumed numerical values given to the distribution parameters, and to the operative and standby units.

Khaled (2008), studied the Cost analysis of a system with preventive maintenance using kolmogorov's forward equation method. The study deals with cost analysis of a two-unit cold standby redundant system with preventive maintenance. The random failure occurs at random time which follow an exponential distribution and also the repair time are assumed to be exponential distributed. Several reliability characteristic are obtained. The mean time to system failure (MTSF) and profit function are studied graphically.

El-Said and El-sherbeny (2010), studied the stochastic analysis of a two unit cold standby system with two stage repair and waiting time. They investigated the cost benefit analysis of a two unit cold standby system with two stage repair of a failed unit. The repair process is divided into two stages, stage one the repairing process of the unit is started but it does not get completed, instead the process is completed in the second stage. The elapsed time between the two stage is called the waiting time. Techniques of regenerative point processes have been used to measure the effectiveness. The dependent availability, steady state availability, reliability,MTTF and profit function were obtained numerically and graphically. The MTTF, the steady state availability and the profit function decreased with respect to the increased of failure rate and waiting rate.

Haggag (2009a), studied the cost analysis of two dissimilar-unit cold standby system with three state and preventive maintenance using linear first order differential equations. He stated that the better maintenance of the system originates better reliability. Also standby support increases the reliability of the system. Approach: Determine the efficacy of preventive maintenance on the reliability and performance of the system. The MTTF, steady state availability and cost analysis of a two dissimilar unit cold standby system with preventive maintenance was discussed. On the assumption that each unit works in three different states; normal, partial failure and total failure. Where the failure and repair time are exponentially distributed. Finally, the result indicated that the better maintenance of parts of the system originated better reliability and performance of the system.

El-Sherbeny *et al* (2009), presented the optimal system for series system with warm standby components and a repairable station. He deals with the reliability and availability characteristics of three different series system configuration with warm standby components and repairable service station. The failure time of the primary and warm standby are assumed to be exponentially distributed with parameter. The breakdown time and the repair time of the service station are also assumed to be exponentially distributed with parameter respectively. They derived the reliability dependent on time, the mean time to failure, and steady state availability for three configurations and perform comparisons.

El-Said and El-Hamid (2008), studied the comparison of reliability characteristics of two systems with preventive maintenance and different modes. He dealt with different behavior of two systems under the assumption that system one works in three different models “Normal, partial failure and Total failure. But system two works in 2 different modes “Normal and Total failure “. The failure time and repair time are exponentially distributed. The two systems go for preventive maintenance randomly (in time). They develop the explicit expression for the mean time system failure MTSF and the steady state availabilities for two systems using linear first order differential equation and perform comparisons theoretically and graphically to observe the effect of preventive maintenance and failure rates on system performance.

Rakesh *et al* (2010), studied the stochastic analysis of a two non-identical unit standby system model. The one unit is considered as priority (P) unit and the other as ordinary (O) unit. The P unit gets priority in operation. A single repair facility appears in and disappears from the system randomly with constant rates. The repair discipline of units is FCFS. The joint distribution of failure and repair times for each unit is taken to be bivariate exponential. Using regenerative point technique various measures of system effective useful to industrial managers are obtained.

Vanderperre (1998), analyses the reliability of Gaver`s parallel sustained by a cold standby unit and attended by two identical repairman. The system satisfies the usual conditions (random variables, perfect repair, instantaneous and perfect switch, queuing). Each operative unit has a constant failure rate but a general time distribution. The reliability analysis is based on a time dependent version of the supplementary variable method. They transform the basic equation in to an integro differential equation of the (mixed) fredholm type. The equation generalizes takack`s integro-differential equation. In order to present computational results, they outline the solution procedure for a repair time distribution with an arbitrary rational laplace- steltjes transform. A particular numerical example display the survivor function with the security interval that ensures a reliability level of at least 95%.

Huairui *et al* (2007), numerous stochastic models for repairable systems have been developed by assuming different time trends, and repair effects. In this paper, a new general repair model based on the repair history is presented. Unlike the existing models, the closed form solutions of the reliability metrics can be easily estimated. The proposed model, as well as the estimation approach, overcomes the draw backs of the existing models. The practical use of the proposed model is demonstrated by a much discussed set of data. Compared to the existing models, the new model is convenient, and provides accurate estimation results.

Jose (1994), studied the cost function for the preventive maintenance replacement problem. Let a discounted continuous review preventive-maintenance be such that its total discounted cost is given by means of two functional equations. They assume that down time is caused by equipment breakdowns and the length of a given downtime is the time necessary to repair the equipment and set it back in operation. The periodic preventive replacement policy is to replace the equipment by new identical equipment when service age is reached, or when the equipment fails.

Sachin and Anand (2009), studied the evaluation of some reliability parameters of a three state repairable system with environmental failure. He dealt with a 3-state repairable complex system with three types of failure. In his paper mathematical model has been developed for exponential failure and general repairs. Various state probabilities have been evaluated in the form of laplace transform. Expression for various reliability parameters of the system are obtained by the inversion process and the computations are done for MTSF and Reliability of the System. All necessary graphical illustrations are given at the end so as to explain the practical utility of the model.

Yusuf (2012), studied two different systems both are requiring supporting unit for their operations. The first system consist of 3-out of-4 subsystem requiring its support from 2-

out-4 subsystem for its operation, while the other system is two unit cold standby where each unit is attached to its supporting unit for its operation. Each system is attached by two repairmen, one repairing the main unit and the other repairing the supporting unit. Explicit expressions for MTSF and Steady- State Availability are developed. He analyze the system by using kolmogrov`s forward equation method. Effects of failure and repair rates on MTSF and steady state availability have also been discussed graphically. Furthermore, some reliability characteristics of the two systems are compared and found that system 1 is better than system 2.

Montri (2009), claims that the research is an investigation of symptoms of tier IV data centre failures incase of unplanned and planned downtimes. The paper examines the consequent impacts of system fault, error and failure from data centre operation. It is important to distinguish among fault, error and failure of the systems. How the syndrome of the active failure and propagating (spread) failure does associated with pervasive causes of disasters (spreading gradually to affect all part of the system) . Each type of systems failure has its own characteristics warning signs. The system availability of the relevant indicators is discussed in some detail, and a comprehensive prevention strategy must take into account of each escalating failure.(System Failure, System Availability and MTBF).

Yusuf and Hussaini (2012), studied the evaluation of reliability and availability characteristics of 2-out of -3 standby systems under a perfect repair condition. Many authors studied the effectiveness of a redundant system under two or three types of failure under the assumption that such failures are repairable. Little attention is paid on whether such repair action can restore the system operating condition to as good as new (perfect repair) and the effect of such perfect repair on the system performance. In this study, various measures of system effectiveness such as mean system time failure(MTSF),steady state availability, busy period and profit function of a 2-out-of-3 repairable system with perfect repair are analyzed using kolmogorov`s forward equation method. Some particular cases have been discussed graphically. The result has indicated that perfect repair action plays vital role on system performance. Simulation show that perfect is important particularly in increasing mean time for system failure, availability and system performance.

Haggag (2009b), studied the cost analysis of k-out of n- repairable system with dependent failure and standby support using kolmogorov`s forward equation method. Many authors have studied k-out of n- repairable system with dependent failure and standby support. The question raised weather repair and standby units support increase the reliability of the system. APPROACH: in the study, the statistical analysis of k-out of n- repairable system with dependent and failure and standby support were discussed. Several reliability characteristics are obtained by using kolmogorov`s forward equation method. After the model is developed a particular case study is discussed to validate the theoretical result, a numerical computation are derived. Tables and graphs have been also given in end. Result, finally indicated that the system with repair and standby support is better than the system without repair and standby support.

However in this research, “Evaluation of reliability and availability characteristics of two different systems, using linear first order differential equation” is studied. Where the second system differs from the first system due to the additional feature of preventive maintenance. The system having one active unit and one warm standby unit with self reset function and maintenance facility. Mean Time – to System Failure, Steady –State

Availability, Busy Period and Profit Function are derived. The evaluations are made theoretically and graphically to observe the effect of preventive maintenance on system performance, and to also observe the effect of failure rate and repair rate on both systems.

CHAPTER THREE

3.0: RESEARCH METHODOLOGY

3.1: Introduction

This research presents reliability and availability analysis of two different systems. Using linear first order differential equations, the second system differs from the first system due to the additional feature of preventive maintenance. Reliability and Availability analysis of system having one active unit and one warm standby unit with self - reset function and one maintenance facility is presented. The failure unit is repaired through self-reset or maintenance according to different failure models. We derived the Mean Time to System Failure (MTSF), Steady State Availability, Busy Period as well as profit function are derived and perform evaluations theoretically and graphically to study the effect of preventive maintenance on system performance.

3.2: Model Description and Assumptions

(1) The system consists of one active unit and one warm standby unit. When the active unit fails, the warm standby unit becomes active without a time delay. One failed unit resets or waits for maintenance and after that it becomes the warm standby unit of the system. When two units fail and need maintenance, the system is in failed state. Only after repairs of two units are completed, will the system start to run again.

(2) All failure rates are constant. There are two kinds of failure. One causes the self-reset of the failed unit the other needs maintenance by a worker.

(3) All failures are statistically independent.

(4) Repair has two forms. One is self-reset and the other is maintenance. A single repair facility is available.

(5) All repair rates are constant.

(6) A repair unit is as good as new.

(7) The switching device's failure rate is zero, and the switch time is zero.

3.3: FIRST TRANSITION SYSTEM

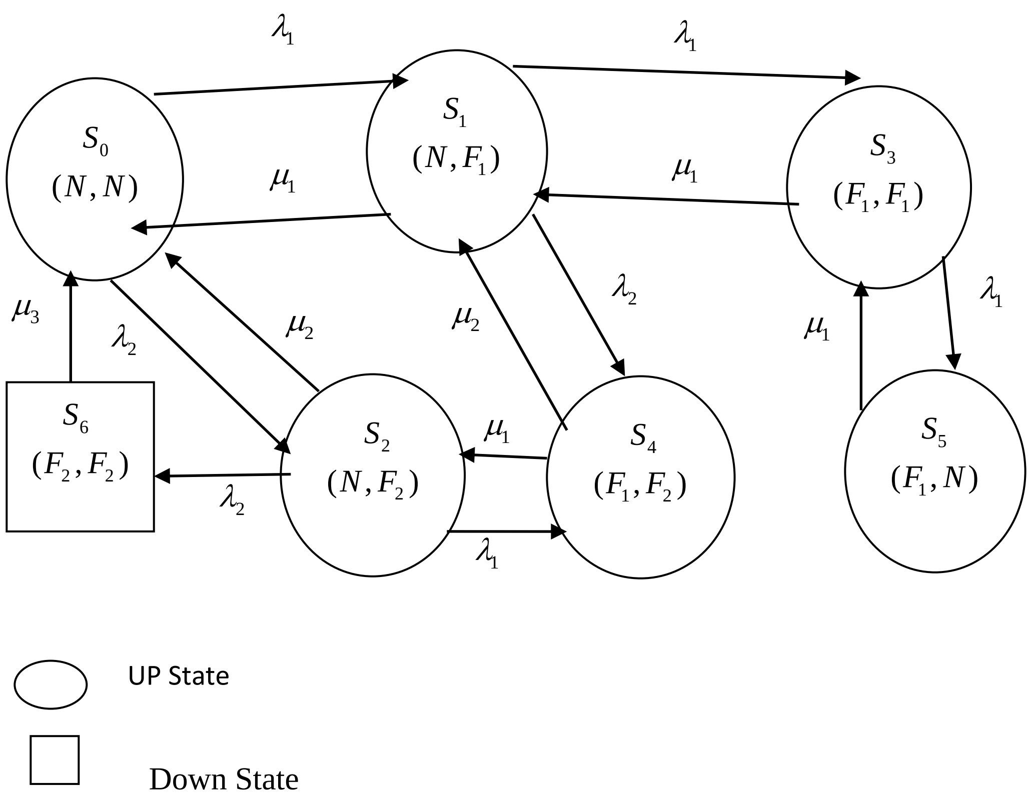


Figure 3.1: State Transition Diagram for the First System

S_0: Initially the system is in full operational state for both the active unit and warm standby unit.

S_1: Secondly, the active unit is normal and the warm standby unit need self reset function and therefore, the system is operational.

S_2: In this state the active unit is normal and warm standby unit need self reset function. Hence the system is operational.

S_3: Both the active unit and warm standby unit need reset-function, and hence the system is operational.

S_4: In this state the active unit needs self-rest function and the warm standby unit need maintenance, the system is also operational.

S_5: The active unit needs self-reset function and warm standby unit is normal, hence the system is operational.

S_6: Both the active unit and warm standby unit need maintenance facility and hence the system is in failed state.

3.4: SECOND TRANSITON SYSTEM

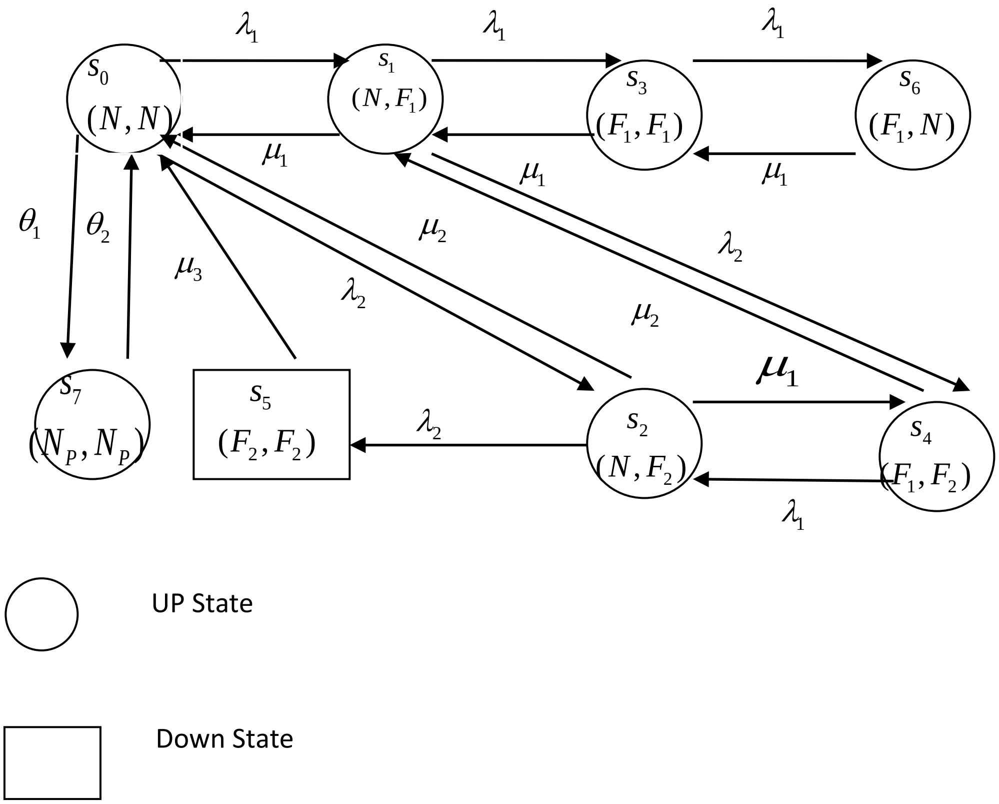


Figure 3. 2: State Transition Diagram for the Second System

S_0: This is an initial state where the active unit and warm standby unit are in full operational state and hence the system is operational.

S_1: The active unit in this state is normal and the warm standby unit needs self-reset function. The system is operational.

S_2: In this state the active unit is normal and warm standby unit need maintenance and hence the system is operational.

S_3: In this state both the active unit and warm standby unit need self-reset function, hence the system is operational.

S_4: In this state the active unit need self-reset function and the warm standby unit is in failure state and need maintenance hence the system is operational.

S_5: In this state both the active unit and the warm standby unit are in failed state and need maintenance, hence the system is in failed state.

S_6: In this state the active unit needs self-reset function and warm standby unit is in normal state and the system is also operational.

S_7: In this state both the active unit and warm standby unit are normal under preventive maintenance.

3.4.1: Mean Time to System Failure ($MTSF_1$)

Let $P_i(t)$ be the probability that the system is in state S_i . If we let P(t) denote the probability row vector at time t, then the initial condition for this problem are

$$P(0) = [p_0(0), p_1(0), p_2(0), p_3(0), p_4(0), p_5(0), p_6(0)] = [1,0,0,0,0,0,0] \qquad (3.1)$$

We obtain the following differential equations

$$\frac{dp_0}{dt} = -(\lambda_1 + \lambda_2)p_0(t) + \mu_1 p_1(t) + \mu_2 p_2(t) + \mu_3 p_6(t)$$
$$\frac{dp_1}{dt} = -(\lambda_1 + \lambda_2 + \mu_1)p_1(t) + \lambda_1 p_0(t) + \mu_1 p_3(t) + \mu_2 p_4(t)$$
$$\frac{dp_2}{dt} = -(\mu_1 + \mu_2 + \lambda_2)p_2(t) + \lambda_1 p_4(t) + \lambda_2 p_0(t)$$
$$\frac{dp_3}{dt} = -(\lambda_1 + \mu_1)p_3(t) + \lambda_1 p_1(t) + \mu_1 p_5(t)$$
$$\frac{dp_4}{dt} = -(\lambda_1 + \mu_2)p_4(t) + \mu_1 p_2(t) + \lambda_2 p_1(t)$$
$$\frac{dp_5}{dt} = -\mu_1 p_5(t) + \lambda_1 p_3(t) \qquad (3.2)$$
$$\frac{dp_6}{dt} = -\mu_3 p_6(t) + \lambda_2 p_2(t)$$

This can be written in the matrix form as

$$\dot{P}_1 = Q_1 \dot{P} \qquad (3.3)$$

where

$$Q_1 = \begin{pmatrix} -(\lambda_1+\lambda_2) & \mu_1 & \mu_2 & 0 & 0 & 0 & \mu_3 \\ \lambda_1 & -(\lambda_1+\lambda_2+\mu_1) & 0 & \mu_1 & \mu_2 & 0 & 0 \\ \lambda_2 & 0 & -(\mu_1+\mu_2+\lambda_2) & 0 & \lambda_1 & 0 & 0 \\ 0 & \lambda_1 & 0 & -(\lambda_1+\mu_1) & 0 & \mu_1 & 0 \\ 0 & \lambda_2 & \mu_1 & 0 & -(\lambda_1+\mu_2) & 0 & 0 \\ 0 & 0 & 0 & \lambda_1 & 0 & -\mu_1 & 0 \\ 0 & 0 & \lambda_2 & 0 & 0 & 0 & -\mu_3 \end{pmatrix} \qquad (3.4)$$

To evaluate the transient solution is too complex. Therefore, we will restrict our selves in calculating the $MTSF_1$. To calculate the $MTSF_1$, we take the transpose matrix of Q and delete the rows and columns for the absorbing states. The new matrix is called A_1. The expected time to reach an absorbing state E_1 is calculated from

$$E_1[T_{P(0)\to P(absorbing)}] = P^*(0)(-A_1^{-1})\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix} \tag{3.5}$$

where

$$A_1 = \begin{bmatrix} -(\lambda_1+\lambda_2) & \lambda_1 & \lambda_2 & 0 & 0 & 0 \\ \mu_1 & -(\lambda_1+\lambda_2+\mu_1) & 0 & \lambda_1 & \lambda_2 & 0 \\ \mu_2 & 0 & -(\mu_1+\mu_2+\lambda_2) & 0 & \mu_1 & 0 \\ 0 & \mu_1 & 0 & -(\lambda_1+\mu_1) & 0 & \lambda_1 \\ 0 & \mu_2 & \lambda_1 & 0 & -(\lambda_1+\mu_2) & 0 \\ 0 & 0 & 0 & \mu_1 & 0 & -\mu_1 \end{bmatrix} \tag{3.6}$$

This method is successful of the following relations:

$$E_1[T_{P(0)\to P(absorbing)}] = P^*(0)\int_0^\infty e^{A_1 t}\,dt \tag{3.7}$$

and

$$\int_0^{\infty} e^{A_1 t} dt = -A_1^{-1}, \quad \textit{Since } A_1 < 0. \tag{3.8}$$

where A_1 is negative definite.

We obtain the following explicit expression for the $MTSF_1$

$$E_1[T_{P(0) \to P(absorbing)}] = MTSF_1 \tag{3.9}$$

3.4.2: Steady – State Availability $A_{T_1}(\infty)$

For the availability case of figure 1, the initial conditions for this problem are the same as for the reliability case,

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0)] = [1, 0, 0, 0, 0, 0, 0]$$

and its matrix form can be expressed as:

$$\begin{bmatrix} \dot{p}_0 \\ \dot{p}_1 \\ \dot{p}_2 \\ \dot{p}_3 \\ \dot{p}_4 \\ \dot{p}_5 \\ \dot{p}_6 \end{bmatrix} = \begin{bmatrix} -(\lambda_1+\lambda_2) & \mu_1 & \mu_2 & 0 & 0 & 0 & \mu_3 \\ \lambda_1 & -(\lambda_1+\lambda_2+\mu_1) & 0 & \mu_1 & \mu_2 & 0 & 0 \\ \lambda_2 & 0 & -(\mu_1+\mu_2+\lambda_2) & 0 & \lambda_1 & 0 & 0 \\ 0 & \lambda_1 & 0 & -(\lambda_1+\mu_1) & 0 & \mu_1 & 0 \\ 0 & \lambda_2 & \mu_1 & 0 & -(\lambda_1+\mu_2) & 0 & 0 \\ 0 & 0 & 0 & \lambda_1 & 0 & -\mu_1 & 0 \\ 0 & 0 & \lambda_2 & 0 & 0 & 0 & -\mu_3 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \end{bmatrix} \tag{3.10}$$

The steady-state availabilities can be obtained using the following procedure. In the steady – state, the derivatives of the state probabilities become zero. That allows us to calculate the steady state availabilities with

$$A_{T_1}(\infty) = 1 - P_6(\infty) \tag{3.11}$$

and

$QP(\infty) = 0$

or, in the matrix form

$$\begin{bmatrix} -(\lambda_1+\lambda_2) & \mu_1 & \mu_2 & 0 & 0 & 0 & \mu_3 \\ \lambda_1 & -(\lambda_1+\lambda_2+\mu_1) & 0 & \mu_1 & \mu_2 & 0 & 0 \\ \lambda_2 & 0 & -(\mu_1+\mu_2+\lambda_2) & 0 & \lambda_1 & 0 & 0 \\ 0 & \lambda_1 & 0 & -(\lambda_1+\mu_1) & 0 & \mu_1 & 0 \\ 0 & \lambda_2 & \mu_1 & 0 & -(\lambda_1+\mu_2) & 0 & 0 \\ 0 & 0 & 0 & \lambda_1 & 0 & -\mu_1 & 0 \\ 0 & 0 & \lambda_2 & 0 & 0 & 0 & -\mu_3 \end{bmatrix} \begin{bmatrix} p_0(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \\ p_5(\infty) \\ p_6(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3.12}$$

To obtain $P_6(\infty)$, we solve (3.I3) and use the following normalizing condition

$$P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + P_5(\infty) + P_6(\infty) = 1 \tag{3.13}$$

We substitute (3.14) in any of the redundant rows in (3.13) to yield

$$\begin{bmatrix} -(\lambda_1+\lambda_2) & \mu_1 & \mu_2 & 0 & 0 & 0 & \mu_3 \\ \lambda_1 & -(\lambda_1+\lambda_2+\mu_1) & 0 & \mu_1 & \mu_2 & 0 & 0 \\ \lambda_2 & 0 & -(\mu_1+\mu_2+\lambda_2) & 0 & \lambda_1 & 0 & 0 \\ 0 & \lambda_1 & 0 & -(\lambda_1+\mu_1) & 0 & \mu_1 & 0 \\ 0 & \lambda_2 & \mu_1 & 0 & -(\lambda_1+\mu_2) & 0 & 0 \\ 0 & 0 & 0 & \lambda_1 & 0 & -\mu_1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} p_0(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \\ p_5(\infty) \\ p_6(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{3.14}$$

The solution of (3.14) provide the steady – state probabilities in the availability case for figure 1, the explicit expression for $A_{T_1}(\infty)$ is given by

$$A_{T_1}(\infty) = 1 - P_6(\infty) \qquad (3.15)$$

3.4.3: Busy Period Analysis (BP_1)

Using the same initial condition as for the reliability case

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0)] = [1,0,0,0,0,0,0] \qquad (3.16)$$

The differential equations can be expressed as

$$\begin{bmatrix} \dot{p}_0 \\ \dot{p}_1 \\ \dot{p}_2 \\ \dot{p}_3 \\ \dot{p}_4 \\ \dot{p}_5 \\ \dot{p}_6 \end{bmatrix} = \begin{bmatrix} -(\lambda_1+\lambda_2) & \mu_1 & \mu_2 & 0 & 0 & 0 & \mu_3 \\ \lambda_1 & -(\lambda_1+\lambda_2+\mu_1) & 0 & \mu_1 & \mu_2 & 0 & 0 \\ \lambda_2 & 0 & -(\mu_1+\mu_2+\lambda_2) & 0 & \lambda_1 & 0 & 0 \\ 0 & \lambda_1 & 0 & -(\lambda_1+\mu_1) & 0 & \mu_1 & 0 \\ 0 & \lambda_2 & \mu_1 & 0 & -(\lambda_1+\mu_2) & 0 & 0 \\ 0 & 0 & 0 & \lambda_1 & 0 & -\mu_1 & 0 \\ 0 & 0 & \lambda_2 & 0 & 0 & 0 & -\mu_3 \end{bmatrix} \begin{bmatrix} p_0(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \\ p_5(\infty) \\ p_6(\infty) \end{bmatrix} \qquad (3.17)$$

In the steady state, the derivatives of the state probabilities become zero and this will

enable us to compute steady- state busy

$$BP_1 = 1 - P_0(\infty) \qquad (3.18)$$

and

$A_1P = 0$

which in matrix form

$$\begin{bmatrix} -(\lambda_1+\lambda_2) & \mu_1 & \mu_2 & 0 & 0 & 0 & \mu_3 \\ \lambda_1 & -(\lambda_1+\lambda_2+\mu_1) & 0 & \mu_1 & \mu_2 & 0 & 0 \\ \lambda_2 & 0 & -(\mu_1+\mu_2+\lambda_2) & 0 & \lambda_1 & 0 & 0 \\ 0 & \lambda_1 & 0 & -(\lambda_1+\mu_1) & 0 & \mu_1 & 0 \\ 0 & \lambda_2 & \mu_1 & 0 & -(\lambda_1+\mu_2) & 0 & 0 \\ 0 & 0 & 0 & \lambda_1 & 0 & -\mu_1 & 0 \\ 0 & 0 & \lambda_2 & 0 & 0 & 0 & -\mu_3 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (3.19)$$

We solve for $P_0(\infty)$

Using the following normalizing condition

$$P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + P_5(\infty) + P_6(\infty) = 1 \quad (3.20)$$

We substitute (3.20) in any of the redundant rows in (3.19) to give

$$\begin{bmatrix} -(\lambda_1+\lambda_2) & \mu_1 & \mu_2 & 0 & 0 & 0 & \mu_3 \\ \lambda_1 & -(\lambda_1+\lambda_2+\mu_1) & 0 & \mu_1 & \mu_2 & 0 & 0 \\ \lambda_2 & 0 & -(\mu_1+\mu_2+\lambda_2) & 0 & \lambda_1 & 0 & 0 \\ 0 & \lambda_1 & 0 & -(\lambda_1+\mu_1) & 0 & \mu_1 & 0 \\ 0 & \lambda_2 & \mu_1 & 0 & -(\lambda_1+\mu_2) & 0 & 0 \\ 0 & 0 & 0 & \lambda_1 & 0 & -\mu_1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (3.21)$$

3.4.4: Profit Function (PF_1)

The expected profit per unit time incurred by the system in the steady – state is given by:

Profit = total revenue from system using – total cost due to repair

$$PF_1 = RA_1(\infty) - CB_1(\infty) \quad (3.22)$$

where PF : is the profit incurred by the system

R : is the revenue per unit up time of the system

C: is the cost per unit time which the system is under repair.

3.4.5: Mean Time to System Failure ($MTSF_2$)

Let $P_i(t)$ be the probability of the second transition system in state S_i. If we let $P^{\#}(t)$ denote the probability row vector at time t, then the initial condition for this problems are

$$P^{\#}(0)=[P_0(0),P_1(0),P_2(0),P_3(0),P_4(0),P_5(0),P_6(0),P_7(0)]=[1,0,0,0,0,0,0,0] \qquad (3.23)$$

we obtain the following differential equations:

$$\begin{aligned}
\frac{dp_0}{dt}&=-(\lambda_1+\lambda_2+\theta_1)p_0(t)+\mu_1p_1(t)+\mu_2p_2(t)+\mu_3p_5(t)+\theta_2p_7(t)\\
\frac{dp_1}{dt}&=-(\lambda_1+\lambda_2+\mu_1)p_1(t)+\lambda_1p_0(t)+\mu_1p_3(t)+\mu_2p_4(t)\\
\frac{dp_2}{dt}&=-(\mu_1+\mu_2+\lambda_2)p_2(t)+\lambda_2p_0(t)+\lambda_1p_4(t)\\
\frac{dp_3}{dt}&=-(\lambda_1+\mu_1)p_3(t)+\lambda_1p_1(t)+\mu_1p_6(t)\\
\frac{dp_4}{dt}&=-(\lambda_1+\mu_2)p_4(t)+\lambda_2p_1(t)+\mu_1p_2(t)\\
\frac{dp_5}{dt}&=-\mu_3p_5(t)+\lambda_2p_2(t)\\
\frac{dp_6}{dt}&=-\mu_1p_6(t)+\lambda_1p_3(t)\\
\frac{dp_7}{dt}&=-\theta_2p_7(t)+\theta_1p_0(t)
\end{aligned} \qquad (3.24)$$

this can be written in the matrix form as

$$\dot{P}_2=Q_2P \qquad (3.25)$$

where

$$Q_2 = \begin{bmatrix} -(\lambda_1+\lambda_2+\theta_1 & \mu_1 & \mu_2 & 0 & 0 & \mu_3 & 0 & \theta_2 \\ \lambda_1 & -(\lambda_1+\lambda_2+\mu_1) & 0 & \mu_1 & \mu_2 & 0 & 0 & 0 \\ \lambda_2 & 0 & -(\mu_1+\mu_2+\lambda_2) & 0 & \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & -(\lambda_1+\mu_1) & 0 & 0 & \mu_1 & 0 \\ 0 & \lambda_2 & \mu_1 & 0 & -(\lambda_1+\mu_2) & 0 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 & 0 & -\mu_3 & 0 & 0 \\ 0 & 0 & 0 & \lambda_1 & 0 & 0 & -\mu_1 & 0 \\ \theta_1 & 0 & 0 & 0 & 0 & 0 & 0 & -\theta_2 \end{bmatrix} \quad (3.26)$$

To evaluate the transient solution is too complex. Therefore, we will restrict ourselves in computing the $MTSF_2$. To evaluate the $MTSF_2$, we take the transpose matrix of Q_2 and delete the rows and columns for the absorbing states. The new matrix is called A_2 the expected time to reach an absorbing state E_2 is evaluated from

$$E_2[T_{P(0)\to P(absorbing)}] = P^{\#*}(0)(-A_2^{-1}) \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad (3.27)$$

in which

$$A_2 = \begin{bmatrix} -(\lambda_1+\lambda_2+\theta_1) & \lambda_1 & \lambda_2 & 0 & 0 & 0 & 0 \\ \mu_1 & -(\lambda_1+\lambda_2+\mu_1) & 0 & \lambda_1 & \lambda_2 & 0 & 0 \\ \mu_2 & 0 & -(\mu_1+\mu_2+\lambda_2) & 0 & \mu_1 & \lambda_2 & 0 \\ 0 & \mu_1 & 0 & -(\lambda_1+\mu_1) & 0 & 0 & \lambda_1 \\ 0 & \mu_2 & \lambda_1 & 0 & -(\lambda_1+\mu_2) & 0 & 0 \\ \mu_3 & 0 & 0 & 0 & 0 & -\mu_3 & 0 \\ 0 & 0 & 0 & \mu_1 & 0 & 0 & -\mu_1 \end{bmatrix} \quad (3.28)$$

This method is successful of the following relations

$$E_2[T_{P(0)\to P(absorbing)}] = P^{\#*}(0)\int_0^{\infty} e^{A_2 t} dt \tag{3.29}$$

and

$$\int_0^{\infty} e^{A_2 t} dt = -A_2^{-1},\ Since\ A_2 < 0 \tag{3.30}$$

where A_2 is negative definite.

We obtain the following explicit expression for the $MTSF_2$

$$E_2[T_{P(0)\to P(\text{absorbing})}] = MTSF_2 \tag{3.31}$$

3.4.6: Steady – State Availability $A_{T_2}(\infty)$

For the availability case of Figure 3.2, the initial conditions for this problem are the same as for the reliability case

$$P^{\#}(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0), P_7(0)] = [1, 0, 0, 0, 0, 0, 0, 0] \tag{3.32}$$

The differential equation can be expressed as

$$\begin{bmatrix} \dot{p}_0 \\ \dot{p}_1 \\ \dot{p}_2 \\ \dot{p}_3 \\ \dot{p}_4 \\ \dot{p}_5 \\ \dot{p}_6 \\ \dot{p}_7 \end{bmatrix} \begin{bmatrix} -(\lambda_1+\lambda_2+\theta_1 & \mu_1 & \mu_2 & 0 & 0 & \mu_3 & 0 & \theta_2 \\ \lambda_1 & -(\lambda_1+\lambda_2+\mu_1) & 0 & \mu_1 & \mu_2 & 0 & 0 & 0 \\ \lambda_2 & 0 & -(\mu_1+\mu_2+\lambda_2) & 0 & \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & -(\lambda_1+\mu_1) & 0 & 0 & \mu_1 & 0 \\ 0 & \lambda_2 & \mu_1 & 0 & -(\lambda_1+\mu_2) & 0 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 & 0 & -\mu_3 & 0 & 0 \\ 0 & 0 & 0 & \lambda_1 & 0 & 0 & -\mu_1 & 0 \\ \theta_1 & 0 & 0 & 0 & 0 & 0 & 0 & -\theta_2 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \end{bmatrix} \quad (3.33)$$

The steady – State availabilities can be obtained using the following procedure. In the Steady – State, the derivatives of the state probabilities vanish. That allows us to calculate the steady – state probabilities with

$$A_{T_2}(\infty) = 1 - P_5(\infty) \quad (3.34)$$

$$\begin{aligned}
\frac{dp_0}{dt} &= -(\lambda_1+\lambda_2+\theta_1)\,p_0(t) + \mu_1 p_1(t) + \mu_2 p_2(t) + \mu_3 p_5(t) + \theta_2 p_7(t) \\
\frac{dp_1}{dt} &= -(\lambda_1+\lambda_2+\mu_1)\,p_1(t) + \lambda_1 p_0(t) + \mu_1 p_3(t) + \mu_2 p_4(t) \\
\frac{dp_2}{dt} &= -(\mu_1+\mu_2+\lambda_2)\,p_2(t) + \lambda_2 p_0(t) + \lambda_1 p_4(t) \\
\frac{dp_3}{dt} &= -(\lambda_1+\mu_1)\,p_3(t) + \lambda_1 p_1(t) + \mu_1 p_6(t) \\
\frac{dp_4}{dt} &= -(\lambda_1+\mu_2)\,p_4(t) + \lambda_2 p_1(t) + \mu_1 p_2(t) \\
\frac{dp_5}{dt} &= -\mu_3 p_5(t) + \lambda_2 p_2(t) \\
\frac{dp_6}{dt} &= -\mu_1 p_6(t) + \lambda_1 p_3(t) \\
\frac{dp_7}{dt} &= -\theta_2 p_7(t) + \theta_1 p_0(t)
\end{aligned} \quad (3.35)$$

and

$$\begin{bmatrix} -(\lambda_1+\lambda_2+\theta_1) & \mu_1 & \mu_2 & 0 & 0 & \mu_3 & 0 & \theta_2 \\ \lambda_1 & -(\lambda_1+\lambda_2+\mu_1) & 0 & \mu_1 & \mu_2 & 0 & 0 & 0 \\ \lambda_2 & 0 & -(\mu_1+\mu_2+\lambda_2) & 0 & \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & -(\lambda_1+\mu_1) & 0 & 0 & \mu_1 & 0 \\ 0 & \lambda_2 & \mu_1 & 0 & -(\lambda_1+\mu_2) & 0 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 & 0 & -\mu_3 & 0 & 0 \\ 0 & 0 & 0 & \lambda_1 & 0 & 0 & -\mu_1 & 0 \\ \theta_1 & 0 & 0 & 0 & 0 & 0 & 0 & -\theta_2 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (3.36)$$

To obtain P_5 we solve (3.36) and use the following normalizing condition

$$P_0(\infty)+P_1(\infty)+P_2(\infty)+P_3(\infty)+P_4(\infty)+P_5(\infty)+P_6(\infty)+P_7(\infty)=1 \quad (3.37)$$

we substitute (3.37) in any one of the redundant rows in (3.36) to yield

$$\begin{bmatrix} -(\lambda_1+\lambda_2+\theta_1) & \mu_1 & \mu_2 & 0 & 0 & \mu_3 & 0 & \theta_2 \\ \lambda_1 & -(\lambda_1+\lambda_2+\mu_1) & 0 & \mu_1 & \mu_2 & 0 & 0 & 0 \\ \lambda_2 & 0 & -(\mu_1+\mu_2+\lambda_2) & 0 & \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & -(\lambda_1+\mu_1) & 0 & 0 & \mu_1 & 0 \\ 0 & \lambda_2 & \mu_1 & 0 & -(\lambda_1+\mu_2) & 0 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 & 0 & -\mu_3 & 0 & 0 \\ 0 & 0 & 0 & \lambda_1 & 0 & 0 & -\mu_1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} p_0(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \\ p_5(\infty) \\ p_6(\infty) \\ p_7(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (3.38)$$

The solution of (3.38) provides the steady – state probabilities in the availability case. For figure 2, the explicit expression for $A_{T_2}(\infty)$ is given by

$$A_{T_2}(\infty)=1-P_5(\infty) \quad (3.39)$$

3.4.7: Busy Period Analysis (BP2)

using

$$P^{\#}(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0), P_7(0)] = [1, 0, 0, 0, 0, 0, 0, 0] \qquad (3.40)$$

the differential equations can be expressed as

$$\begin{aligned}
\frac{dp_0}{dt} &= -(\lambda_1 + \lambda_2 + \theta_1)p_0(t) + \mu_1 p_1(t) + \mu_2 p_2(t) + \mu_3 p_5(t) + \theta_2 p_7(t) \\
\frac{dp_1}{dt} &= -(\lambda_1 + \lambda_2 + \mu_1)p_1(t) + \lambda_1 p_0(t) + \mu_1 p_3(t) + \mu_2 p_4(t) \\
\frac{dp_2}{dt} &= -(\mu_1 + \mu_2 + \lambda_2)p_2(t) + \lambda_2 p_0(t) + \lambda_1 p_4(t) \\
\frac{dp_3}{dt} &= -(\lambda_1 + \mu_1)p_3(t) + \lambda_1 p_1(t) + \mu_1 p_6(t) \\
\frac{dp_4}{dt} &= -(\lambda_1 + \mu_2)p_4(t) + \lambda_2 p_1(t) + \mu_1 p_2(t) \\
\frac{dp_5}{dt} &= -\mu_3 p_5(t) + \lambda_2 p_2(t) \qquad (3.41) \\
\frac{dp_6}{dt} &= -\mu_1 p_6(t) + \lambda_1 p_3(t) \\
\frac{dp_7}{dt} &= -\theta_2 p_7(t) + \theta_1 p_0(t)
\end{aligned}$$

and in matrix form as

$$\begin{bmatrix} \dot{p}_0 \\ \dot{p}_1 \\ \dot{p}_2 \\ \dot{p}_3 \\ \dot{p}_4 \\ \dot{p}_5 \\ \dot{p}_6 \\ \dot{p}_7 \end{bmatrix} = \begin{bmatrix}
-(\lambda_1 + \lambda_2 + \theta_1 & \mu_1 & \mu_2 & 0 & 0 & \mu_3 & 0 & \theta_2 \\
\lambda_1 & -(\lambda_1 + \lambda_2 + \mu_1) & 0 & \mu_1 & \mu_2 & 0 & 0 & 0 \\
\lambda_2 & 0 & -(\mu_1 + \mu_2 + \lambda_2) & 0 & \lambda_1 & 0 & 0 & 0 \\
0 & \lambda_1 & 0 & -(\lambda_1 + \mu_1) & 0 & 0 & \mu_1 & 0 \\
0 & \lambda_2 & \mu_1 & 0 & -(\lambda_1 + \mu_2) & 0 & 0 & 0 \\
0 & 0 & \lambda_2 & 0 & 0 & -\mu_3 & 0 & 0 \\
0 & 0 & 0 & \lambda_1 & 0 & 0 & -\mu_1 & 0 \\
\theta_1 & 0 & 0 & 0 & 0 & 0 & 0 & -\theta_2
\end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \end{bmatrix} \qquad (3.42)$$

In the steady-state, the derivatives of the state probabilities become zero and this will enable us to compute steady-state busy

$$BP_2 = 1 - P_0(\infty) \tag{3.43}$$

$$\begin{aligned}
\frac{dp_0}{dt} &= -(\lambda_1 + \lambda_2 + \theta_1)p_0(t) + \mu_1 p_1(t) + \mu_2 p_2(t) + \mu_3 p_5(t) + \theta_2 p_7(t) \\
\frac{dp_1}{dt} &= -(\lambda_1 + \lambda_2 + \mu_1)p_1(t) + \lambda_1 p_0(t) + \mu_1 p_3(t) + \mu_2 p_4(t) \\
\frac{dp_2}{dt} &= -(\mu_1 + \mu_2 + \lambda_2)p_2(t) + \lambda_2 p_0(t) + \lambda_1 p_4(t) \\
\frac{dp_3}{dt} &= -(\lambda_1 + \mu_1)p_3(t) + \lambda_1 p_1(t) + \mu_1 p_6(t) \\
\frac{dp_4}{dt} &= -(\lambda_1 + \mu_2)p_4(t) + \lambda_2 p_1(t) + \mu_1 p_2(t) \\
\frac{dp_5}{dt} &= -\mu_3 p_5(t) + \lambda_2 p_2(t) \\
\frac{dp_6}{dt} &= -\mu_1 p_6(t) + \lambda_1 p_3(t) \\
\frac{dp_7}{dt} &= -\theta_2 p_7(t) + \theta_1 p_0(t)
\end{aligned} \tag{3.44}$$

and

$A_2P = 0$, which in matrix form

$$\begin{bmatrix}
-(\lambda_1+\lambda_2+\theta_1) & \mu_1 & \mu_2 & 0 & 0 & \mu_3 & 0 & \theta_2 \\
\lambda_1 & -(\lambda_1+\lambda_2+\mu_1) & 0 & \mu_1 & \mu_2 & 0 & 0 & 0 \\
\lambda_2 & 0 & -(\mu_1+\mu_2+\lambda_2) & 0 & \lambda_1 & 0 & 0 & 0 \\
0 & \lambda_1 & 0 & -(\lambda_1+\mu_1) & 0 & 0 & \mu_1 & 0 \\
0 & \lambda_2 & \mu_1 & 0 & -(\lambda_1+\mu_2) & 0 & 0 & 0 \\
0 & 0 & \lambda_2 & 0 & 0 & -\mu_3 & 0 & 0 \\
0 & 0 & 0 & \lambda_1 & 0 & 0 & -\mu_1 & 0 \\
\theta_1 & 0 & 0 & 0 & 0 & 0 & 0 & -\theta_2
\end{bmatrix}
\begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3.45}$$

We solve for $P_0(\infty)$. Using the following normalizing condition

$$P_0(\infty)+P_1(\infty)+P_2(\infty)+P_3(\infty)+P_4(\infty)+P_5(\infty)+P_6(\infty)+P_7(\infty)=1 \tag{3.46}$$

We substitute (3.45) in any of the redundant rows (3.44) to give

$$\begin{bmatrix} -(\lambda_1+\lambda_2+\theta_1 & \mu_1 & \mu_2 & 0 & 0 & \mu_3 & 0 & \theta_2 \\ \lambda_1 & -(\lambda_1+\lambda_2+\mu_1) & 0 & \mu_1 & \mu_2 & 0 & 0 & 0 \\ \lambda_2 & 0 & -(\mu_1+\mu_2+\lambda_2) & 0 & \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & -(\lambda_1+\mu_1) & 0 & 0 & \mu_1 & 0 \\ 0 & \lambda_2 & \mu_1 & 0 & -(\lambda_1+\mu_2) & 0 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 & 0 & -\mu_3 & 0 & 0 \\ 0 & 0 & 0 & \lambda_1 & 0 & 0 & -\mu_1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} p_0(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \\ p_5(\infty) \\ p_6(\infty) \\ p_7(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{3.47}$$

.

3.4.8: Profit Function (PF_2)

The expected profit per unit time incurred by the system in the steady state is given by

Profit = total revenue from system using – total cost due to repair

$$PF_2=RA_2(\infty)-CB_2(\infty) \tag{3.48}$$

where

PF: is the profit incurred to the system

R: is the revenue per unit up time of the system

C: is the cost per unit time which the system is under repair.

CHAPTER FOUR

4.0: RESULT AND DISCUSSION

4.1: Introduction

This chapter describes the result obtained from chapter three by using MATLAB. It shows that the result of the Mean – Time to System Failure, Steady- State Availability, Profit Function and Busy Period by the use of MATLAB.

4.1.1: Mean-Time to System Failure ($MTSF_1$)

To calculate the $MTSF_1$, we take the transpose matrix of Q and the delete the rows and columns for the absorbing states.

$$E_1[T_{P(0)\rightarrow(absorbing)}] = MTSF_1 = \frac{a_1}{b_1} \tag{4.1}$$

where

$$\begin{aligned} a_1 = {} & \mu_1\lambda_2\mu_1^2[(\mu_2\mu_1^2 + \lambda_1\lambda_2\mu_1 + \mu_2\lambda_2\mu_1 + \lambda_1\mu_2\mu_1 + \mu_2^2\mu_1 + \lambda_1\lambda_2^2 + \lambda_1\mu_2\lambda_2) + \mu_2\lambda_2\mu_1 + \lambda_1\mu_2\mu_1 + \lambda_1^2\lambda_2 \\ & + \lambda_1\mu_2\lambda_2 + \lambda_1\mu_2^2 + \lambda_1^2\mu_2] + \mu_1\mu_1^2\lambda_2^2(\lambda_1\mu_1 + \mu_2\mu_1 + \lambda_1\lambda_2 + \lambda_1^2) + \lambda_1\lambda_2\mu_1^2(\mu_2\lambda_2\mu_1 + \lambda_1\mu_2\mu_1 + \lambda_1^2\lambda_2 + \lambda_1 \\ & \mu_2\lambda_2 + \lambda_1\mu_2^2 + \lambda_1^2\mu_2) + \mu_1\mu_1^2\lambda_2^2(\mu_1^2 + \lambda_2\mu_1 + \lambda_1\mu_1 + \lambda_1\lambda_2 + \lambda_1\mu_2) + \lambda_1^2\mu_1\lambda_2(\mu_2\lambda_2\mu_1 + \lambda_1\mu_2\mu_1 + \lambda_1^2\lambda_2 \\ & + \lambda_1\mu_2\lambda_2 + \lambda_1\mu_2^2 + \lambda_1^2\mu_2) \end{aligned} \tag{4.2}$$

and

$$b_1 = \mu_1\lambda_2\mu_1^2\lambda_2^2(\lambda_1\mu_1 + \mu_2\mu_1 + \lambda_1\lambda_2 + \lambda_1^2) \tag{4.3}$$

4.1.2: Steady State Availability $A_{T_1}(\infty)$

To obtain the steady state availability, the steady state can be obtained in the following procedure. After obtaining the differential equation the derivatives of the state probabilities become zero. That allows us to compute the Steady – State probabilities with,

$$
\begin{aligned}
&A_{T_1}(\infty) = 1 - P_6(\infty) \\
&\textit{and} \\
&QP(\infty) = 0 \\
&\textit{where } P_6(\infty) \textit{ is the failed state.}
\end{aligned}
$$

After taking or using the normalization condition, one will see that the sum of the probabilities becomes one. Availability is the sum of the operational state.

$AV = SSA(0) + SSA(1) + SSA(2) + SSA(3) + SSA(4) + SSA(5)$. Or $A_{T_1}(\infty)$ *is given by*

$$A_{T_1}(\infty) = 1 - P_6(\infty) = 1 - \frac{\Delta_1}{\Delta} \tag{4.4}$$

where

$$
\begin{aligned}
D_1 = {} & \mu_1^2\mu_3(\mu_2\mu_1^2 + \lambda_1\lambda_2\mu_1 + \mu_2\lambda_2\mu_1 + \lambda_1\mu_2\mu_1 + \mu_2^2\mu_1 + \lambda_1\lambda_2^2 + \lambda_1\mu_2\lambda_2) + \mu_1^2\mu_3(\mu_2\lambda_2\mu_1 + \lambda_1\mu_2\mu_1 + \lambda_1^2\lambda_2 \\
& + \lambda_1\mu_2\lambda_2 + \lambda_1\mu_2^2 + \lambda_1^2\mu_2) + \lambda_2\mu_1^2\mu_3(\lambda_1\mu_1 + \mu_2\mu_1 + \lambda_1\lambda_2 + \lambda_1^2) + \lambda_1\mu_1\mu_3(\mu_2\lambda_2\mu_1 + \lambda_1\mu_2\mu_1 + \lambda_1^2\lambda_2 + \lambda_1\mu_2\lambda_2 \\
& + \lambda_1\mu_2^2 + \lambda_1^2\mu_2) + \lambda_2\mu_1^2\mu_3(\mu_1^2 + \lambda_2\mu_1 + \lambda_1\mu_1 + \lambda_1\lambda_2 + \lambda_1\mu_2) + \lambda_1^2\mu_3(\mu_2\lambda_2\mu_1 + \lambda_1\mu_2\mu_1 + \lambda_1^2\lambda_2 + \lambda_1\mu_2\lambda_2 \\
& + \lambda_1\mu_2^2 + \lambda_1^2\mu_2)
\end{aligned} \tag{4.5}
$$

and

$$
\begin{aligned}
D = {} & \lambda_1^4\lambda_2\mu_3 + \lambda_1^4\mu_2\mu_3 + \lambda_1^3\mu_2\lambda_2\mu_3 + \lambda_1^3\lambda_2\mu_1\mu_3 + \lambda_1^3\mu_2^2\mu_3^2 + 2\lambda_1^3\mu_2\mu_1\mu_3 + \lambda_1^2\mu_1^2\lambda_2^2 + 2\lambda_1^2\mu_2\lambda_2\mu_1\mu_3 + \\
& 2\lambda_1^2\lambda_2\mu_1^2\mu_3 + \lambda_1^2\mu_2^2\mu_1\mu_3 + 2\lambda_1^2\mu_1^2\mu_2\mu_3 + \lambda_1\lambda_2^3\mu_1^2 + 3\lambda_1\lambda_2^2\mu_1^2\mu_3 + \lambda_1\lambda_2^2\mu_1^3 + 4\lambda_1\mu_1^2\lambda_2\mu_2\mu_3 \\
& + 3\lambda_1\mu_1^3\mu_3\lambda_2 + \lambda_1\mu_1^2\mu_2^2\mu_3 + 2\lambda_1\mu_1^3\mu_2\mu_3 + \mu_2\lambda_2^2\mu_1^3 + \lambda_2^2\mu_1^3\mu_3 + 3\mu_1^3\lambda_2\mu_2\mu_3 + \lambda_2\mu_1^4\mu_3 + \mu_1^3\mu_2^2\mu_3 \\
& + \mu_1^4\mu_2\mu_3
\end{aligned} \tag{4.6}
$$

4.1.3: Busy Period Analysis (BP_1)

To obtain the Busy Period, using the initial condition of the first system the derivative of the state become zero and this will enable us to compute Steady- State Busy.

$$BP_1 = 1 - P_0(\infty)$$

$$BP_1(\infty) = 1 - \frac{\mu_1^2\mu_3(\mu_2\mu_1^2 + \lambda_1\lambda_2\mu_1 + \mu_2\lambda_2\mu_1 + \lambda_1\mu_2\mu_1 + \mu_2^2\mu_1 + \lambda_1\lambda_2^2 + \lambda_1\mu_2\lambda_2)}{\begin{array}{c}(\lambda_1^4\lambda_2\mu_3 + \lambda_1^4\mu_2\mu_3 + \lambda_1^3\mu_2\lambda_2\mu_3 + \lambda_1^3\lambda_2\mu_1\mu_3 + \lambda_1^3\mu_2^2\mu_3 + 2\lambda_1^3\mu_2\mu_1\mu_3 + \lambda_1^2\mu_1^2\lambda_2^2 + 2\lambda_1^2\mu_2\lambda_2\mu_1\mu_3 + \\ 2\lambda_1^2\lambda_2\mu_1^2\mu_3 + \lambda_1^2\mu_2^2\mu_1\mu_3 + 2\lambda_1^2\mu_1^2\mu_2\mu_3 + \lambda_1\lambda_2^3\mu_1^2 + 3\lambda_1\lambda_2^2\mu_1^2\mu_3 + \lambda_1\lambda_2^2\mu_1^3 + 4\lambda_1\mu_1^2\lambda_2\mu_2\mu_3 + 3\lambda_1\mu_1^3 \\ \mu_3\lambda_2 + \lambda_1\mu_1^2\mu_2^2\mu_3 + 2\lambda_1\mu_1^3\mu_2\mu_3 + \mu_2\lambda_2^2\mu_1^3 + \lambda_2^2\mu_1^3\mu_3 + 3\mu_1^3\lambda_2\mu_2\mu_3 + \lambda_2^3\mu_1^4\mu_3 + \mu_1^3\mu_2^2\mu_3 + \mu_1^4\mu_2\mu_3)\end{array}} \quad (4.7)$$

4.1.4: Mean-Time to System Failure ($MTSF_2$)

The mean-time to system failure of the second system

$$MTSF_2 = \frac{a_2}{b_2} \quad (4.8)$$

$$\begin{aligned}a_2 &= \theta_2\mu_1\lambda_2\mu_1^2(\mu_2\mu_1^2 + \lambda_1\lambda_2\mu_1 + \lambda_1\mu_2\mu_1 + \mu_2\lambda_2\mu_1 + \mu_2^2\mu_1 + \lambda_1\mu_2\lambda_2 + \lambda_1\lambda_2^2) + \theta_2\lambda_2\mu_1^2(\mu_2\lambda_2\mu_1 + \\ &\lambda_1\mu_2\mu_1 + \lambda_1^2\mu_2 + \lambda_1\mu_2^2) + \theta_2\lambda_2\mu_1^2(\mu_2\lambda_2\mu_1 + \lambda_1\mu_2\mu_1 + \lambda_1^2\mu_2 + \lambda_1\mu_2^2) + \theta_2\mu_1\mu_1^2\lambda_2^2(\mu_1^2 + \lambda_2\mu_1 + \lambda_1\mu_1 + \lambda_1\mu_2 + \lambda_1\lambda_2) \\ &+\theta_2\lambda_2\lambda_1^2\mu_1(\mu_2\lambda_2\mu_1 + \lambda_1\mu_2\mu_1 + \lambda_1^2\mu_2 + \lambda_1\mu_2^2) + \theta_2\mu_1^2\mu_1\lambda_2^2(\lambda_1\mu_1 + \mu_2\mu_1 + \lambda_1\lambda_2 + \lambda_1^2) + \theta_1\mu_1\lambda_2\mu_1^2 \\ &(\lambda_1\mu_1 + \mu_2\mu_1 + \lambda_1\lambda_2 + \lambda_1^2)[\mu_2\mu_1 + \lambda_1\lambda_2\mu_1 + \lambda_1\mu_2\mu_1 + \mu_2\lambda_2\mu_1 + \mu_2^2\mu_1 + \lambda_1\mu_2\lambda_2 + \lambda_1\lambda_2]\end{aligned} \quad (4.9)$$

and

$$b_2 = \theta_2\mu_1\lambda_2\mu_1^2\lambda_2^2(\lambda_1\mu_1 + \mu_2\mu_1 + \lambda_1\lambda_2 + \lambda_1^2) \quad (4.10)$$

4.1.5: Steady- State Availability $A_{T_2}(\infty)$

The State Availability can be obtained using the initial condition of the second system. The derivatives of the state probabilities vanish and that allow us to calculate the steady-state probabilities.

AV=SSA(0)+SSA(1)+SSA(2)+SSA(3)+SSA(4)+SSA(6)+SSA(7). Or,

$$A_{T_2}(\infty) = 1 - P_5(\infty) = 1 - \frac{\Delta_2}{\Delta_0} \tag{4.11}$$

where

$$\begin{aligned} D_2 = {} & \mu_1^2\mu_3\theta_2(\mu_2\mu_1^2 + \lambda_1\lambda_2\mu_1 + \lambda_1\mu_2\mu_1 + \mu_2\lambda_2\mu_1 + \mu_2^2\mu_1 + \lambda_1\mu_2\lambda_2 + \lambda_1\lambda_2^2) + \mu_1^2\mu_3\theta_2(\mu_2\lambda_2\mu_1 + \lambda_1\mu_2\mu_1 + \\ & \lambda_1^2\lambda_2 + \lambda_1\mu_2\lambda_2 + \lambda_1^2\mu_2 + \lambda_1\mu_2^2) + \mu_1^2\lambda_2\mu_3\theta_2(\lambda_1\mu_1 + \mu_2\mu_1 + \lambda_1\lambda_2 + \lambda_1^2) + \mu_1\mu_3\theta_2\lambda_1(\mu_2\lambda_2\mu_1 + \lambda_1\mu_2\mu_1 + \\ & \lambda_1^2\lambda_2 + \lambda_1\mu_2\lambda_2 + \lambda_1^2\mu_2 + \lambda_1\mu_2^2) + \mu_1^2\lambda_2\mu_3\theta_2(\mu_1^2 + \lambda_2\mu_1 + \lambda_1\mu_1 + \lambda_1\mu_2 + \lambda_1\lambda_2) + \mu_3\theta_2\lambda_1^2(\mu_2\lambda_2\mu_1 + \lambda_1 \\ & \mu_2\mu_1 + \lambda_1^2\lambda_2 + \lambda_1\mu_2\lambda_2 + \lambda_1^2\mu_2 + \lambda_1\mu_2^2) + \mu_3\mu_1^2\theta_1(\mu_2\mu_1 + \lambda_1\lambda_2\mu_1 + \lambda_1\mu_2\mu_1 + \mu_2\lambda_2\mu_1 + \mu_2^2\mu_1 + \lambda_1\mu_2\lambda_2 + \\ & \lambda_1\lambda_2) \end{aligned} \tag{4.12}$$

and

$$\begin{aligned} D_0 = {} & \mu_1^3\mu_3\theta_1\mu_2\lambda_2 + \lambda_1^3\mu_2\lambda_2\mu_3\theta_2 + \lambda_1^4\mu_2\mu_3\theta_2 + \lambda_1^4\mu_3\lambda_2\theta_2 + \lambda_1^3\mu_3\mu_2^2\theta_2 + \mu_1^4\mu_2\mu_3\theta_2 + \mu_1^4\theta_1\mu_2\mu_3 + \mu_1^4\mu_3\lambda_2 \\ & \theta_2 + \mu_1^3\lambda_1\lambda_2^2\theta_2 + \mu_1^3\lambda_2^2\mu_3\theta_2 + \mu_1^3\mu_3\mu_2^2\theta_1 + \mu_1^3\mu_3\mu_2^2\theta_2 + \mu_1^3\mu_2\lambda_2^2\theta_2 + \mu_1^2\lambda_1^2\lambda_2^2\theta_2 + \mu_1^2\lambda_1\lambda_2^3\theta_2 + 3\mu_1^3\lambda_1\mu_3\lambda_2\theta_2 \\ & + \mu_1^3\lambda_1\theta_1\mu_2\mu_3 + \mu_1^3\lambda_1\mu_3\theta_1\lambda_2 + 2\mu_1^3\lambda_1\mu_2\mu_3\theta_2 + 3\mu_1^3\mu_2\lambda_2\mu_3\theta_2 + 2\mu_1^2\lambda_1^2\mu_3\lambda_2\theta_2 + 2\mu_1^2\lambda_1^2\mu_2\mu_3\theta_2 + \mu_1^2\lambda_1\mu_3\mu_2^2 \\ & \theta_2 + 3\mu_1^2\lambda_1\lambda_2^2\mu_3\theta_2 + \mu_1^2\lambda_1\mu_3\lambda_2^2\theta_1 + 4\mu_1^2\lambda_1\mu_2\lambda_2\mu_3\theta_2 + \mu_1^2\lambda_1\mu_2\lambda_2\mu_3\theta_1 + \mu_1\lambda_1^3\mu_3\lambda_2^2\theta_2 + 2\mu_1\lambda_1^3\mu_2\mu_3\theta_2 + \mu_1\lambda_1^2 \\ & \mu_3\mu_2^2\theta_2 + 2\mu_1\lambda_1^2\mu_2\lambda_2\mu_3\theta_2 \end{aligned} \tag{4.13}$$

4.1.6: Busy Period Analysis (BP_2)

The Busy Period Analysis can be obtained as

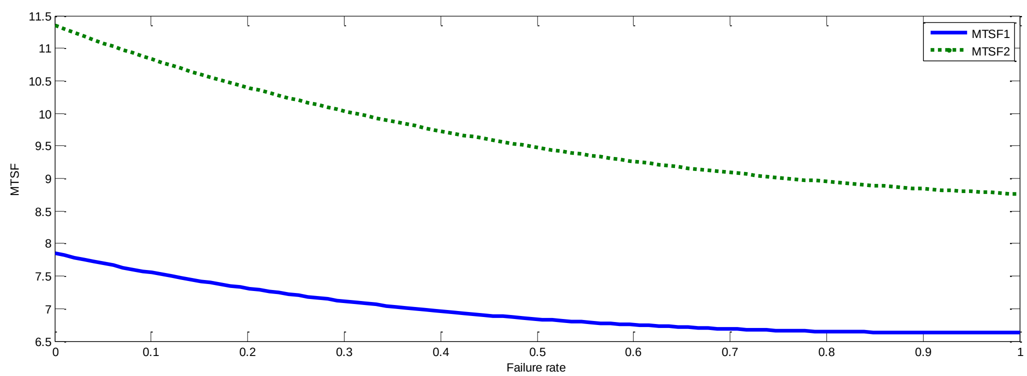
$$BP_2 = 1 - SSA(0), Or$$
$$BP_2(\infty) = 1 - P_0^{\#}(\infty)$$

$$B_2(\infty) = 1 - \frac{\mu_1^2\mu_3\theta_2(\mu_2\mu_1^2 + \lambda_1\lambda_2\mu_1 + \lambda_1\mu_2\mu_1 + \mu_2\lambda_2\mu_1 + \mu_2^2\mu_1 + \lambda_1\mu_2\lambda_2 + \lambda_1\lambda_2^2)}{\begin{array}{l}\mu_1^3\mu_3\theta_1\mu_2\lambda_2 + \lambda_1^3\mu_2\lambda_2\mu_3\theta_2 + \lambda_1^4\mu_2\mu_3\theta_2 + \lambda_1^4\mu_3\lambda_2\theta_2 + \lambda_1^3\mu_3\mu_2^2\theta_2 + \mu_1^4\mu_2\mu_3\theta_2 + \mu_1^4\theta_1\mu_2\mu_3 + \mu_1^4\mu_3\lambda_2 \\ \theta_2 + \mu_1^3\lambda_1\lambda_2^2\theta_2 + \mu_1^3\lambda_2^2\mu_3\theta_2 + \mu_1^3\mu_3\mu_2^2\theta_1 + \mu_1^3\mu_3\mu_2^2\theta_2 + \mu_1^3\mu_2\lambda_2^2\theta_2 + \mu_1^2\lambda_1^2\lambda_2^2\theta_2 + \mu_1^2\lambda_1\lambda_2^3\theta_2 + 3\mu_1^3\lambda_1\mu_3\lambda_2\theta_2 \\ + \mu_1^3\lambda_1\theta_1\mu_2\mu_3 + \mu_1^3\lambda_1\mu_3\theta_1\lambda_2 + 2\mu_1^3\lambda_1\mu_2\mu_3\theta_2 + 3\mu_1^3\mu_2\lambda_2\mu_3\theta_2 + 2\mu_1^2\lambda_1^2\mu_3\lambda_2\theta_2 + 2\mu_1^2\lambda_1^2\mu_2\mu_3\theta_2 + \mu_1^2\lambda_1\mu_3\mu_2^2 \\ \theta_2 + 3\mu_1^2\lambda_1\lambda_2^2\mu_3\theta_2 + \mu_1^2\lambda_1\mu_3\lambda_2^2\theta_1 + 4\mu_1^2\lambda_1\mu_2\lambda_2\mu_3\theta_2 + \mu_1^2\lambda_1\mu_2\lambda_2\mu_3\theta_1 + \mu_1\lambda_1^3\mu_3\lambda_2^2\theta_2 + 2\mu_1\lambda_1^3\mu_2\mu_3\theta_2 + \mu_1\lambda_1^2 \\ \mu_3\mu_2^2\theta_2 + 2\mu_1\lambda_1^2\mu_2\lambda_2\mu_3\theta_2\end{array}} \qquad (4.14)$$

To observe the effect of preventive maintenance on system behavior, taking for $\lambda_1 and \mu_1$ and keeping the other parameters fixed at

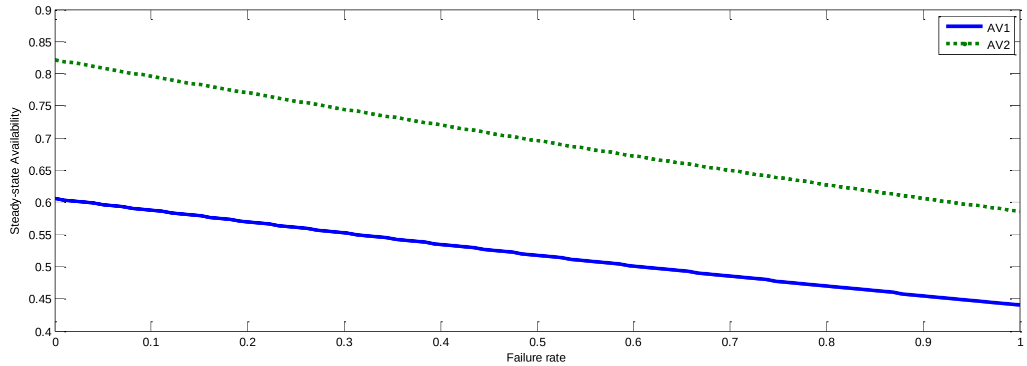
$$\mu_1 = 0.02, \mu_2 = 0.002, \mu_3 = 0.3, \lambda_2 = 0.1, \theta_1 = 0.001, \theta_2 = 0.2 \ and$$
$$\lambda_1 = 0.01, \lambda_2 = 0.002, \mu_2 = 0.02, \mu_3 = 0.1, \theta_1 = 0.001, \theta_2 = 0.003, R = 1000, C = 100$$

We use computer to evaluate two configurations in terms of their $MTSF_j$, $AT_j(\infty)$ *and* $PF_j(\infty)$ .Where j=1,2.



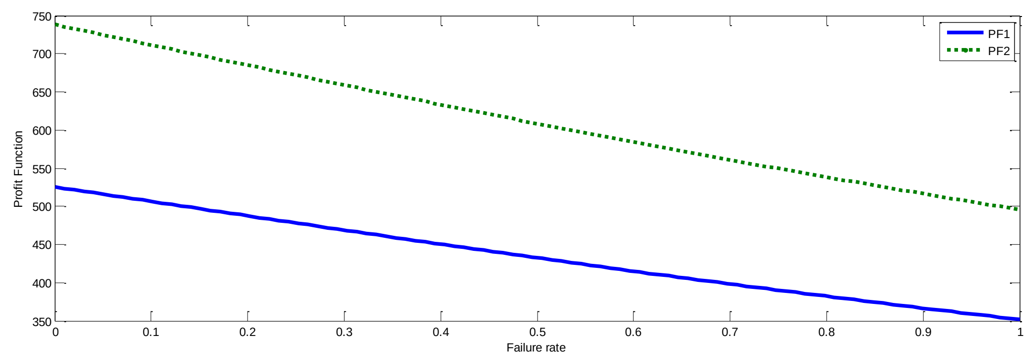
Failure Rate (λ)

Figure 4.1: Relationship between Failure Rate and MTSF



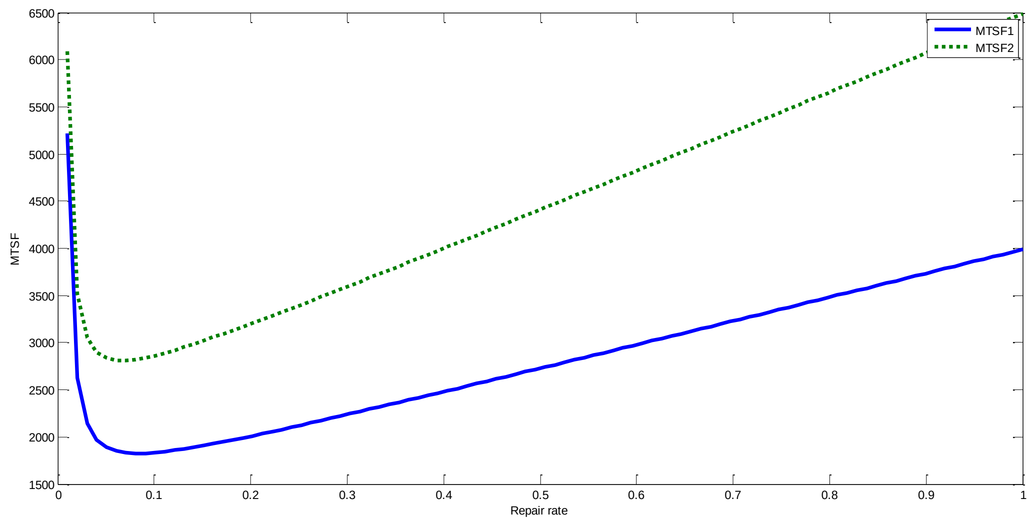
Failure Rate (λ)

Figure 4.2: Relationship between the Failure Rate and the Steady- State Availability



Failure Rate (λ)

Figure 4.3: Relationship between the Failure Rate and Profit Function.



Repair Rate

Figure 4.4: Relationship between Repair Rate and MTSF.

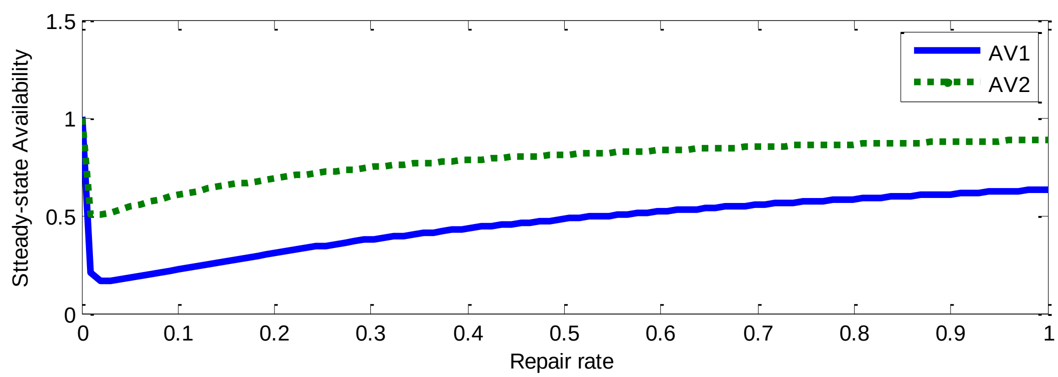


Figure 4.5: Relationship between Repair Rate and Steady-State Availability.

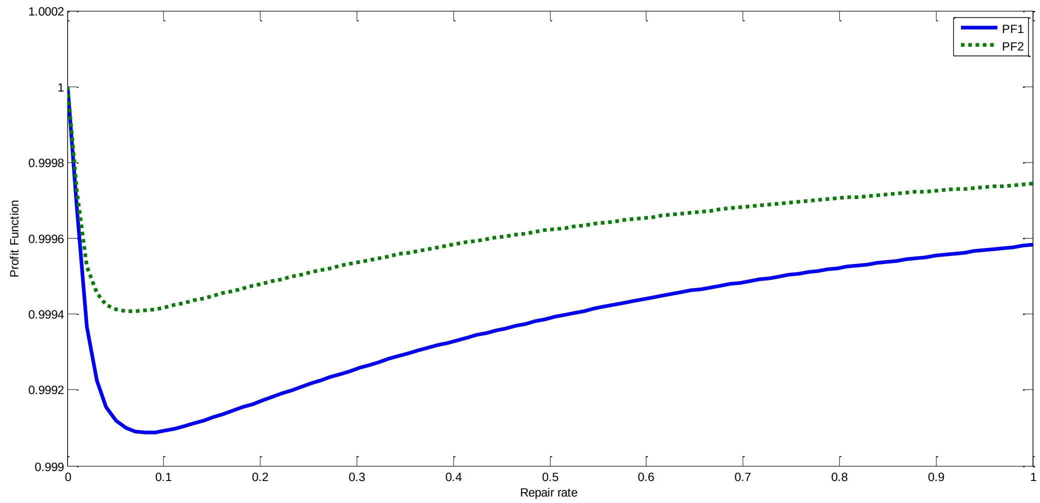


Figure 4.6: Relationship between Repair Rate and Profit Function.

4.2: Discussion of Result

By evaluating the MTSF with respect to failure rate λ_1 of figure 4.1 theoretically and graphically, it was observing that increase in failure rate λ_1 at constant $\mu_1 = 0.02, \mu_2 = 0.002, \mu_3 = 0.3, \lambda_2 = 0.1, \theta_1 = 0.001, \theta_2 = 0.2, R = 1000, C = 100$, the MTSF of the system decreases for both systems with preventive maintenance and without preventive maintenance. The result shows that, the MTSF with preventive maintenance is longer than the system without preventive maintenance that is to say the system with preventive maintenance is better than the system without preventive maintenance. However, the Steady-State Availability and Profit Function with respect to failure rate λ_1 of figure 4.2 and 4.3 are decreasing as a result of increase in failure rate, but second system have more reliability than the first system due to the additional feature of preventive maintenance. Moreover, by evaluating the MTSF with respect to repair rate (μ_1) theoretically and graphically, the result shows that increase in repair rate at constant $\lambda_2 = 0.002, \lambda_1 = 0.01, \mu_2 = 0.02, \mu_3 = 0.1, \theta_1 = 0.001, \theta_2 = 0.003, R = 1000, C = 100$, the MTSF, Steady State Availability and Profit Function of figure 4.4,4.5 and 4.6 are increasing as a result of increase in repair rate . It concluded that the system with preventive maintenance is better than the system without preventive maintenance. Therefore, the second system is more reliable than the first system due to the additional feature of preventive maintenance.

CHAPTER FIVE

5.0: SUMMARY AND CONCLUSSION

5.1: Summary

This chapter summarizes the work done in the research. The research is a study of reliability and availability characteristics of two different system using system of linear fist order ordinary differential equation. Two systems were considered, where the second system differs from the first system due to addition feature of preventive maintenance. Each system consisting of one active unit and one warn stand-by unit with self reset function and maintenance facility. MTSF, Steady-State Availability, Busy Period Analysis and Profit Function were found by the use of MATLAB**.**

5.2: Conclusion

In this study the performance of two systems were evaluated using Linear First Order Differential Equation. Where the second system differs from the first system due to the additional feature of preventive maintenance. The result shows that increase in failure rate lead to the decrease in MTSF, Steady State Availability and Profit Function of figure 4.1, 4.2 and 4.3. Also increase in repair rate lead to the increase in MTSF, Steady State Availability and Profit Function of figure 4.4, 4.5 and 4.6 then, it was observing that second system has better reliability due to the additional feature of preventive maintenance.

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